SM3: 3.3: Graphing Rational Functions

Warm-up: For each function, state the x-values of the vertical asymptotes (VA), holes (H), and end behaviors (EB):

1) $f(x) = \dfrac{x-2}{x+3}$

VA: $x = -3$

H: $\emptyset$

EB: $y = 1$

2) $f(x) = \dfrac{(x-1)}{(x+5)(x-1)}$

VA: $x = -5$

H: $x = 1$

EB: $y = 0$

3) $f(x) = \dfrac{(x+2)^2}{x+2}$

VA: $\emptyset$

H: $x = -2$

EB: oblique

4) $f(x) = \dfrac{1}{(x-6)(x+7)}$

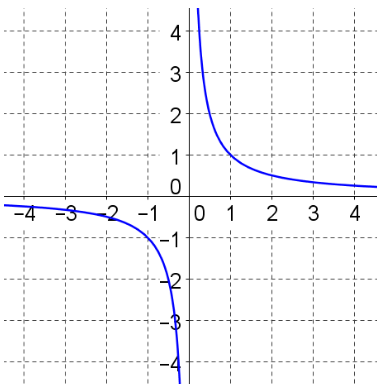
VA: $x = \{-7, 6\}$

H: $\emptyset$

EB: $y = 0$

Problems: Describe the asymptotic and end behavior(s) using limit notation.

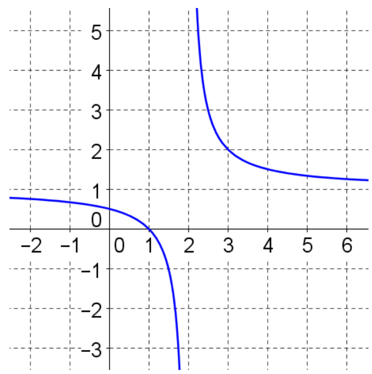
5)



$\lim_{x\to 0^-} f(x) = -\infty, \lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 0, \lim_{x\to \infty} f(x) = 0$

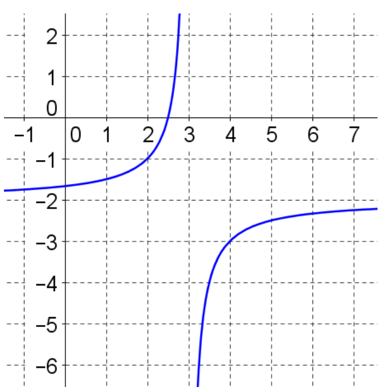
6)



$\lim_{x\to 2^-} f(x) = -\infty, \lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 1, \lim_{x\to \infty} f(x) = 1$

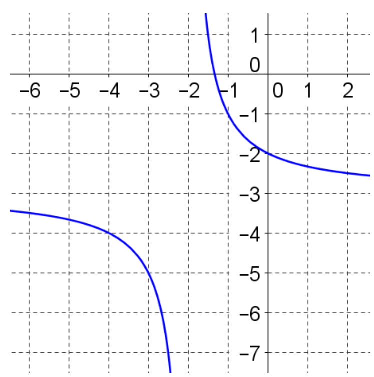
7)



$\lim_{x\to 3^-} f(x) = \infty, \lim_{x\to 3^+} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = -2, \lim_{x\to \infty} f(x) = -2$

8)



$\lim_{x\to -2^-} f(x) = -\infty, \lim_{x\to -2^+} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = -3, \lim_{x\to \infty} f(x) = -3$

Simplify the functions (be sure to include stipulations); state the values of the vertical asymptotes (VA), holes (H), and end behaviors (EB):

9) $f(x) = \frac{x^2 + 2x + 1}{x^2 + 4x + 3}$

$f(x) = \frac{(x+1)^2}{(x+1)(x+3)}$

$f(x) = \frac{(x+1)}{(x+3)}; x \neq -1, -3$

VA: $x = -3$

H: $x = -1$

EB: $y = 1$

10) $f(x) = \frac{2x^2 - 5x - 12}{x^3 - 16x}$

$f(x) = \frac{(2x+3)(x-4)}{x(x+4)(x-4)}$

$f(x) = \frac{(2x+3)}{x(x+4)}; x \neq 4, 0, -4$

VA: $x = \{-4, 0\}$

H: $x = 4$

EB: $y = 0$

11) $f(x) = \frac{12x^2 - 5x - 2}{9x^2 - 12x + 4}$

$f(x) = \frac{(4x+1)(3x-2)}{(3x-2)^2}$

$f(x) = \frac{(4x+1)}{(3x-2)}; x \neq \frac{2}{3}$

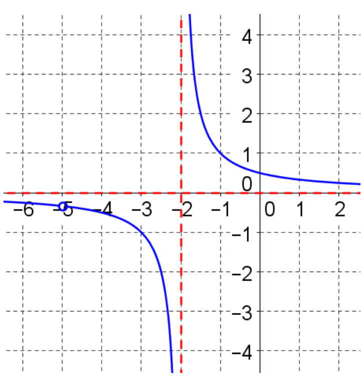
VA: $x = \frac{2}{3}$

H: $\emptyset$

EB: $y = \frac{4}{3}$

Simplify and sketch the function (use dashed lines for vertical asymptotes and open points for holes); describe the vertically and horizontally asymptotic behavior(s) of the function using limit notation:

12) $f(x) = \frac{x+5}{x^2 + 7x + 10}$



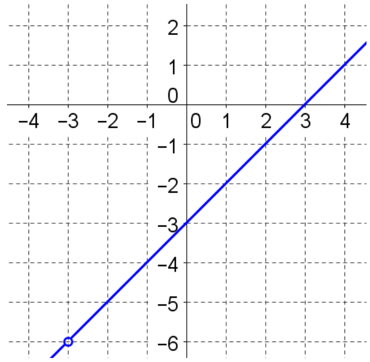
$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = 0$

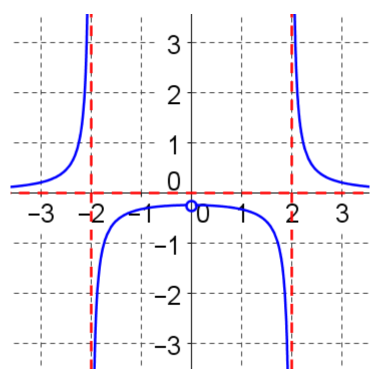
$\lim_{x \to \infty} f(x) = 0$

13) $f(x) = \frac{x^2 - 9}{x + 3}$



$f(x)$ has no vertical or horizontal asymptotes.

14) $f(x) = \frac{x}{x^3 - 4x}$



$\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = -\infty$

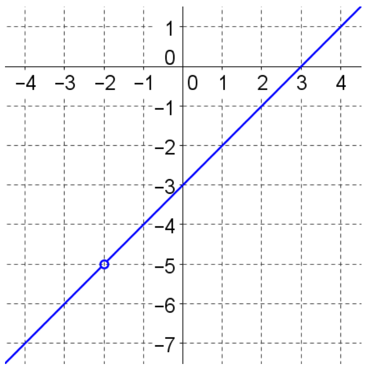
$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = 0$

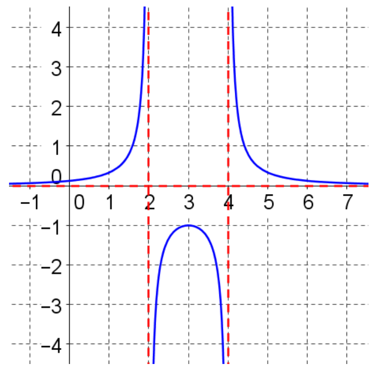
$\lim_{x \to \infty} f(x) = 0$

15) $f(x) = \dfrac{x^2 - x - 6}{x + 2}$



$f(x)$ has no vertical or horizontal asymptotes.

16) $f(x) = \dfrac{1}{x^2 - 6x + 8}$



$\lim_{x \to 2^-} f(x) = \infty$

$\lim_{x \to 2^+} f(x) = -\infty$

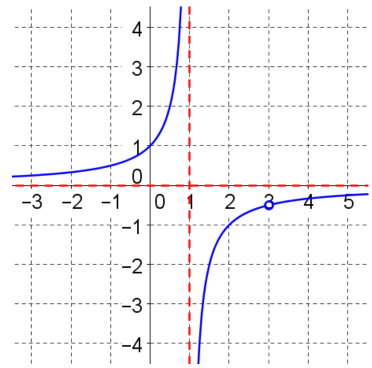
$\lim_{x \to 4^-} f(x) = -\infty$

$\lim_{x \to 4^+} f(x) = \infty$

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

17) $f(x) = \dfrac{-(x-3)}{x^2 - 4x + 3}$



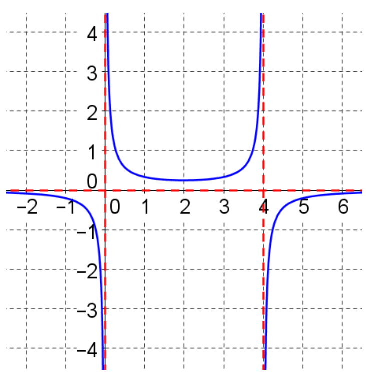
$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

18) $f(x) = -\dfrac{1}{x^2 - 4x}$



$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = \infty$

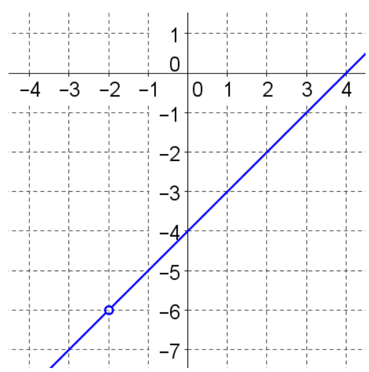
$\lim_{x \to 4^-} f(x) = \infty$

$\lim_{x \to 4^+} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = 0$

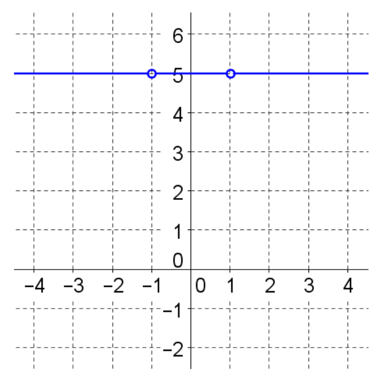
$\lim_{x \to -\infty} f(x) = 0$

19) $f(x) = \dfrac{x^2 - 2x - 8}{x + 2}$



$f(x)$ has no vertical or horizontal asymptotes.

20) $f(x) = \dfrac{5x^2 - 5}{x^2 - 1}$



$f(x)$ has no vertical or horizontal asymptotes.

Problem Creation: Graph functions that exhibits the following properties:

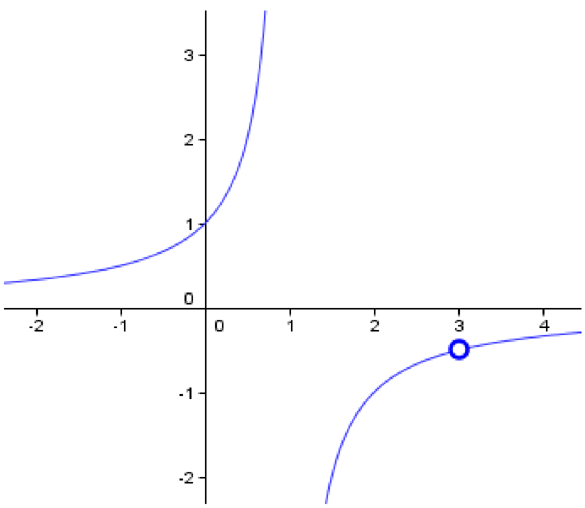
21) Sketch $f(x)$, as described below:

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$

$f(x)$ is strictly increasing

$D_f = (-\infty, 1) \cup (1,3) \cup (3, \infty)$



22) Sketch $g(x)$, as described below:

$\lim_{x \to -2^-} g(x) = \infty$; $\lim_{x \to 2^-} g(x) = -\infty$

$\lim_{x \to -2^+} g(x) = -\infty$; $\lim_{x \to 2^+} g(x) = \infty$

$g(x)$ has even symmetry

$D_g = (-\infty, -2) \cup (-2,0) \cup (0,2) \cup (2, \infty)$